1.35, 1.42, and 1; and the line represents calculation from Eq. (2.2) at $Sh_1 > 0.25$]. Here $Sh_1 = k\Delta x$ ($k = 2\pi f/u_2$ is the wave number). The data from experiments at $Sh_1 > 0.25$ are described satisfactorily by $R' = \exp[-(a_x/b_x)Sh_1]\cos(a_x Sh_1)$ ($a_x = 1.57$, $b_x = 3.14$).

The nature of the relation $R' = R'(Sh_1)$ corresponds to hydrodynamic pressure fluctuations. Indeed, the velocity of sound is characteristic of acoustic pressure fluctuations as $Sh_1 \to 0$, $R' \to 1$. In our case, the determining velocity is the flow velocity u_2 and at $Sh_1 < 0.25$ a decrease in Sh_1 at $\Delta x/\ell = \text{const}$ causes a decrease in R' (see Fig. 5), which is inherent to hydrodynamic pressure fluctuations.

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INTERNAL WAVE FIELD IN THE NEIGHBORHOOD OF A FRONT EXCITED BY A SOURCE MOVING OVER A SMOOTHLY VARYING BOTTOM

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UDC 551.466

The problem of the propagation of surface waves harmonic in time and quasisinusoidal in space over a smoothly varying bottom is solved in [1] by using the geometric optics method. An analogous problem for internal waves with an arbitrary Brunt–Väisälä frequency distribution over the depth was examined in [2]. The case of internal waves locally sinusoidal in space and time in the presence of slowly varying shear flows was investigated in [3]. Airey wave transformation in a smoothly inhomogeneous layer along the horizontal is examined in [4]. Fronts and lines of equal phase are constructed in [5] for a source moving in a stratified fluid layer in the case of constant layer depth. The asymptotic of the solution for the moving source in the neighborhood of the front of a mode taken separately was written down in [6].

The problem of an internal wave field in the neighborhood of the front of a separate mode generated by a point mass source moving over a smoothly varying bottom is examined in this paper by the method of traveling waves [7], which is one modification of the geometric optics method.

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1. FORMULATION OF THE PROBLEM AND SELECTION OF THE FORM OF THE SOLUTION

Let us consider a fluid layer with the Brunt–Väisälä frequency N(z) bounded by a surface z = 0 and the bottom z = H(X, Y). A point source of intensity Q moves uniformly and rectilinearly with velocity V at a depth z_0 in the positive direction of the X axis. Then the velocity field in the Boussinesq approximation will satisfy the following linearized system of equations:

$$\frac{\partial^2}{\partial T^2}\left(\Delta w+\frac{\partial^2 w}{\partial z^2}\right)+N^2(z)\,\Delta w=Q\delta''_{TT}(X-VT)\,\delta(Y)\,\delta'(z-z_0),$$
$$\Delta\mathbf{u}+\nabla\frac{\partial w}{\partial z}=Q\delta(z-z_0)\,\nabla(\delta(X-VT)\,\delta(Y)). \tag{1.1}$$

Here $\nabla = (\partial/\partial X, \partial/\partial Y)$, $\Delta = \partial^2/\partial X^2 + \partial^2/\partial Y^2$; w is the vertical velocity component; $\mathbf{u} = (u_1, u_2)$ is the horizontal velocity vector. The nonpenetration conditions

$$w=0 \text{ for } z=0,\ w=\mathbf{u}\cdot\nabla H(X,Y) \text{ for } z=H(X,Y). \tag{1.2}$$

are assumed satisfied on the layer boundaries.

Let us introduce the dimensionless parameter $\varepsilon = \lambda/L \ll 1$ that characterizes the smoothness of the change in depth of the bottom; λ is the characteristic wavelength; and L is the horizontal scale of the change in depth of the bottom. Then, in the "slow variables" $x = \varepsilon X$, $y = \varepsilon Y$, $t = \varepsilon T$ (the slowness of the change in z is not assumed), the motion equations (1.1) and the boundary conditions (1.2) are written in the form

$$\frac{\partial^2}{\partial t^2}\left(\varepsilon^2\Delta w+\frac{\partial^2 w}{\partial z^2}\right)+N^2(z)\,\Delta w=\varepsilon^2 Q\delta''_{tt}(x-Vt)\,\delta(y)\,\delta'(z-z_0), \tag{1.3}$$

$$\varepsilon\Delta\mathbf{u}+\nabla\frac{\partial w}{\partial z}=\varepsilon^2 Q\delta(z-z_0)\,\nabla(\delta(x-Vt)\,\delta(y));$$
$$w=0 \text{ for } z=0,\ w=\varepsilon\mathbf{u}\cdot\nabla h(x,y) \text{ for } z=h(x,y). \tag{1.4}$$

The solution in the case of a layer of constant depth h is presented for w in [6] as the sum of the modes $w=\sum_{n=1}^{\infty} w_n$. The first term of the asymptotic is written down there for w_n near the front expressed in terms of the Airey function derivative whose argument depends on the first two coefficients of the expansion of the dispersion curve $k_n(\omega) = c_n^{-1}\omega + d_n\omega^3 + \ldots$ at zero, where $k_n(\omega)$ is the eigennumber of the spectral problem

$$F''_{nzz}(z,\omega)+k_n^2(\omega)\left[\frac{N^2(z)}{\omega^2}-1\right]F_n(z,\omega)=0,\quad F_n(0,\omega)=F_n(h,\omega)=0. \tag{1.5}$$

We shall also seek the solution of the system (1.3), (1.4) in the form of a sum of modes $w=\sum_{n=1}^{\infty} w_n$, $\mathbf{u}=\sum_{n=1}^{\infty}\mathbf{u}_n$. Later we refer all the computations to a mode taken separately by omitting the subscript n. Starting from the above, as well as from the structure of the asymptotic of the solution in a layer of constant depth [6], we will seek the solution of the system (1.3), (1.4) in the form

$$w=\varepsilon^{2/3}\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\varepsilon^{(2/3)(k+i)}w_{ik}v_i(\varphi),\quad \mathbf{u}=\varepsilon^{1/3}\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\varepsilon^{(2/3)(k+i)}\mathbf{u}_{ik}v_{i+1}(\varphi) \tag{1.6}$$

[$v_i'(\varphi) = v_{i-1}(\varphi)$, $v_0(\varphi) = \mathrm{Ai}'(\varphi)$ is the Airey function derivative; $\varphi = \varepsilon^{-2/3}((t - \tau(x, y))\cdot\sigma(x, y))$, where we consider the argument φ of the order of one].

Since we will be interested in only the first term of the asymptotic for w, we then rewrite (1.6)

$$w=\varepsilon^{2/3}A(z,x,y)v_0(\varphi)+\varepsilon^{4/3}(B(z,x,y)v_0(\varphi)+$$
$$+C(z,x,y)v_1(\varphi))+O(\varepsilon^2),\ \mathbf{u}=\varepsilon^{1/3}\mathbf{u}_0(z,x,y)v_1(\varphi)+O(\varepsilon). \tag{1.7}$$

The functions $A(z, x, y)$, $u_0(z, x, y)$, $\tau(x, y)$, $\sigma(x, y)$ are to be determined. Substituting (1.7) into the second equation of (1.3) and equating terms for ε^0, we have $u_0 = A_z'(z, x, y)\nabla\tau(x,y)/(\sigma(x,y)|\nabla\tau(x,y)|^2)$. We find the boundary conditions for the functions A, B, C by substituting (1.7) into (1.4): $A = B = C = 0$ for $z = 0$; $A = B = 0$, $C = A_z'\nabla\tau\nabla h/(\sigma|\nabla\tau|^2)$ for $z = h(x, y)$.

2. DERIVATION OF THE FUNDAMENTAL EQUATIONS

Let us turn to finding the equations for the functions $\tau(x, y)$, $A(z, x, y)$, and $\sigma(x, y)$. Substituting (1.7) into the first equation of (1.3) and equating terms of the order of $\varepsilon^{-2/3}$ we obtain

$$A''_{zz}(z, x, y) + |\nabla\tau(x, y)|^2 N^2(z) A(z, x, y) = 0,\ A(0, x, y) = A(h(x, y), x, y) = 0. \tag{2.1}$$

Let us note that the eigenfunctions $A(z, x, y)$ are determined from (2.1) to the accuracy of an arbitrary factor dependent only on x and y; consequently, it is convenient to represent the function $A(z, x, y)$ in the form $A(z, x, y) = \psi(x, y)/f(z, x, y)$, where $f(z, x, y)$ is a solution of the spectral problem (2.1) and satisfies the normalization condition

$$\int_0^{h(x,y)} N^2(z) f^2(z, x, y)\, dz = 1. \tag{2.2}$$

The eigenfunctions $f(z, x, y)$ and numbers $\lambda(x, y)$ of the problem (2.1) are assumed known. Then we have the eikonal equation for $\tau(x, y)$:

$$\left(\frac{\partial\tau}{\partial x}\right)^2 + \left(\frac{\partial\tau}{\partial y}\right)^2 = \lambda^2(x, y). \tag{2.3}$$

To find the functions $\psi(x, y)$ and $\sigma(x, y)$ we equate terms of the order ε^0 after substitution of (1.7) into (1.3). Using the equality $v_0^{IV}(\varphi) = -\varphi v_0'' - 3v_0'$ we obtain two equations (in B containing terms with v_0'', and in C containing terms with v_0'):

$$\sigma^2(B''_{zz} + \lambda^2 N^2(z) B) = 2\varphi A N^2(z)\nabla\sigma\nabla\tau + \varphi A\sigma^4\lambda^2, \quad B = 0 \text{ for } z = 0, h(x, y); \tag{2.4}$$

$$\sigma^2(C''_{zz} + \lambda^2 N^2(z) C) = 2\sigma N^2(z)\nabla A\nabla\tau + A N^2(z)(2\nabla\sigma\nabla\tau + \sigma\Delta\tau) + 3A\sigma^4\lambda^2, \tag{2.5}$$

$$C = 0 \text{ for } z = 0, \quad C = A_z'\nabla\tau\nabla h/(\sigma\lambda^2) \text{ for } z = h(x, y).$$

Let us first examine Eq. (2.4). Multiplying both its sides by the function $A(z, x, y)$ and integrating with respect to z between 0 and $h(x, y)$, we find the equation for σ:

$$2\nabla\sigma\nabla\tau + a(x, y)\lambda^2\sigma^4 = 0 \quad \left(a(x, y) = \int_0^{h(x,y)} f^2(z, x, y)\, dz\right). \tag{2.6}$$

It can be shown that the functions $a(x, y)$ and $\lambda(x, y)$ are expressed in terms of the expansion of the dispersion curve $k(\omega, x, y) = c^{-1}(x, y)\omega + d(x, y)\omega^3 + \ldots$ of the spectral problem (1.5) at zero, in which in place of the functions $F(z, \omega)$ and $k(\omega)$ there are $F(z, \omega, x, y)$ and $k(\omega, x, y)$, while the variables x and y are considered fixed:

$$\lambda(x, y) = c^{-1}(x, y),\ a(x, y) = 2d(x, y)c(x, y),$$

where $c(x, y)$ is the group velocity for $\omega = 0$: $c(x, y) = [\partial k(\omega, x, y)/\partial\omega]_{\omega=0}^{-1}$.

Let us examine (2.5). We multiply both its sides by $A(z, x, y)$ and integrate with respect to z between 0 and $h(x, y)$. Taking account of the normalization condition (2.2), we obtain

$$-\sigma\lambda^{-2}\psi^2[f_z'(h, x, y)]^2\nabla\tau\nabla h = \sigma\nabla\tau\nabla\psi^2 + \psi^2(2\nabla\tau\nabla\sigma + \sigma\Delta\tau) + 3\psi^2\sigma^4\lambda^2 a. \tag{2.7}$$

Differentiating (2.1) with respect to the horizontal variable, it is easy to show that $[f_z'(h, x, y)]^2 \nabla h(x, y) = -\nabla\lambda^2(x, y)$. Then we rewrite the transport equations (2.7) in the form

$$\nabla \ln\left(\frac{\psi^2}{\lambda^2\sigma^4}\right)\nabla\tau + \Delta\tau = 0. \tag{2.8}$$

Therefore, the construction of the field w (1.7) reduces to solving the eikonal equation (2.3) and the transport equations (2.6) and (2.8).

3. SOLUTION OF THE EIKONAL AND TRANSPORT EQUATIONS

The characteristic system for (2.3) (see [8], say) appears as follows ($p = \partial\tau/\partial x$, $q = \partial\tau/\partial y$):

$$\dot{x} = c^2(x, y)\,p, \quad \dot{y} = c^2(x, y)\,q, \quad \dot{p} = -c_x'/c(x, y), \quad \dot{q} = -c_y'/c(x, y). \tag{3.1}$$

There hence results that $\dot{\tau} = 1$; consequently, it is convenient to take the eikonal τ as the parameter of integration. A solution of the system (3.1) is the one-parameter family of functions $x(\tau, \tau_0)$, $y(\tau, \tau_0)$, $p(\tau, \tau_0)$, $q(\tau, \tau_0)$, whose first two functions determine rays on the x, y plane, and τ_0 is the initial eikonal or, equivalently, the time of ray emergence from the source. We assume the source moves along the axis $y = 0$ and passes the origin at the time $\tau = 0$. Then we have initial conditions for the system (3.1):

$$x_0 = V\tau_0,\ y_0 = 0,\ p_0 = 1/V;\ q_0 = \pm\sqrt{1/c^2(x_0, 0) - 1/V^2}. \tag{3.2}$$

The ray equations $x = x(\tau, \tau_0)$, $y = y(\tau, \tau_0)$ for fixed τ_0 yield a specific ray and a wave front for fixed τ. We assume that the ray equations are solvable for τ and τ_0:

$$\tau = \tau(x, y),\ \tau_0 = \tau_0(x, y). \tag{3.3}$$

For this it is necessary that the Jacobian $D \equiv x_\tau' y_{\tau_0}' - x_{\tau_0}' y_\tau' \neq 0$. Equations (3.3) for the point x, y determine the eikonal τ (the time of front arrival at the point x, y) and the initial eikonal τ_0 (the time of ray emergence from the source).

Transport equations (2.6) and (2.8) are integrated along the characteristics of (3.1). The appropriate quadrature for (2.6) has the form

$$\sigma(x, y) = \left[\frac{3}{2}\int\limits_{\tau_0(x,y)}^{\tau(x,y)} a(x(t, \tau_0), y(t, \tau_0))\,dt\right]^{-1/3}. \tag{3.4}$$

Taking account of the expression along the ray [8], $\Delta\tau = \nabla \ln(J/c)\nabla\tau$ [$J(x, y)$ is the geometric divergence of a ray tube ($J = D/c$)], integration of (2.8) yields the "conservation law" $c(x, y)\psi^2(x, y)J(x, y)/(\sigma^4(x, y)J(x_0, 0)) = B(x_0)$. Here $J(x, y)$ and $J(x_0, 0)$ are the geometric divergence of the ray tube at the front and at the point of ray emergence, respectively, $J(x_0, 0) = \sqrt{V^2 - c^2(x_0, 0)}$. The constant $B(x_0)$ is found from the solution of the problem with constant depth of the bottom $h(x_0, 0)$: $B(x_0) = Qc^3(x_0, 0)f_z'(z_0, x_0, 0)/[4(V^2 - c^2(x_0, 0))]$. We write down the final expression

$$\psi(x, y) = \frac{Q\sigma^2(x, y)\left(V^2 - c^2(x_0, 0)\right)^{1/2} c^{3/2}(x_0, 0)\, f_z'(z_0, x_0, 0)}{2c^{1/2}(x, y)\, J^{1/2}(x, y)}. \tag{3.5}$$

Therefore, we have the following scheme for finding the vertical velocity field in the neighborhood of a front of a moving source: a) we solve the characteristic system (3.1) with the initial conditions (3.2); b) solving the ray equations, we find the eikonal $\tau(x, y)$ and the time of ray emergence $\tau_0(x, y)$; c) solving the boundary-value problem (2.1), we obtain the normalized eigenfunction $f(z, x, y)$ and the coefficient $a(x, y)$; d) integrating $a(x, y)$ along a ray, we determine $\sigma(x, y)$ (3.4); e) we find the geometric divergence J, say, by numerical differentiation; f) evaluating the function $\psi(x, y)$ by means of (3.5) and multiplying by $f(z, x, y)$ we have the amplitude $A(z, x, y)$; g) multiplying the amplitude $A(z, x, y)$ by the Airey function derivative of argument φ, we obtain the vertical velocity of a mode taken separately.

4. EXAMPLE

Let us consider the case when the Brunt–Väisälä frequency N = const and the depth of the bottom depends only on one coordinate in a linear manner $H(y) = \beta y$. Let us introduce a coordinate system with the x axis proceeding along the "shore" ($y = 0$), a source moves from left to right in the positive direction of the x axis at the velocity V parallel to the "shore" at a distance y_0 away and at a depth z_0. Let us examine the first mode. Then (2.1) yields the following eigenfunction $f(z, y)$ and eigennumber $\lambda(y)$ ($\gamma = N\beta/\pi$):

$$f(z, y) = \frac{\sqrt{2}}{N\sqrt{\beta y}} \sin\frac{\pi z}{\beta y}, \quad \lambda(y) \equiv \frac{1}{c(y)} = \frac{1}{\gamma y}. \tag{4.1}$$

Let us write down the characteristic system and the initial conditions for the eikonal equation

$$\dot{x} = \gamma^2 y^2/V, \; x_0 = V\tau_0, \; \dot{y} = \pm\gamma y\sqrt{1 - (\gamma y/V)^2}, \; y_0 = y_0. \tag{4.2}$$

Here and henceforth, the upper sign corresponds to the domain $y > y_0$ and the lower to the domain $y < y_0$.

Integrating system (4.2), we obtain the ray equation

$$y = \frac{V}{\gamma}\operatorname{ch}^{-1}\left(\pm\operatorname{arch}\left(\frac{V}{\gamma y_0}\right) - \gamma(\tau - \tau_0)\right), \quad x = x_0 + \frac{\gamma}{V} y_0 y \operatorname{sh}(\gamma(\tau - \tau_0)) \tag{4.3}$$

[$\operatorname{arch} x = \ln(x + \sqrt{x^2 - 1})$]. The rays given by system (4.3) are semicircles of radius V/γ with centers located along the "shore." These semicircles have an envelope (caustic) for $y = V/\gamma$. Henceforth, the field outside the caustic circle and the "shore" is examined.

Since the wave pattern in this case is stationary in a coordinate system moving together with the source ($\xi = Vt - x$), then the front is determined from the equations

$$\frac{d\xi}{dy} = \frac{\pm\sqrt{V^2 - (\gamma y)^2}}{\gamma y}, \quad \xi(y_0) = 0 \tag{4.4}$$

and has the form

$$\xi = \pm\frac{V}{\gamma}(\alpha_1(y) - \alpha_2(y)),$$
$$\alpha_1(y) = \operatorname{arch}\left(\frac{V}{\gamma y_0}\right) - \operatorname{arch}\left(\frac{V}{\gamma y}\right), \quad \alpha_2(y) = \sqrt{1 - \left(\frac{\gamma y_0}{V}\right)^2} - \sqrt{1 - \left(\frac{\gamma y}{V}\right)^2}. \tag{4.5}$$

The ray equations (4.3) are solved for τ and τ_0:

$$\tau = \frac{x}{V} \pm \frac{1}{\gamma}(\alpha_1(y) - \alpha_2(y)), \quad \tau_0 = \frac{x}{V} \mp \frac{1}{\gamma}\alpha_2(y).$$

The coefficient is $a(x, y) = N^{-2}$; hence,

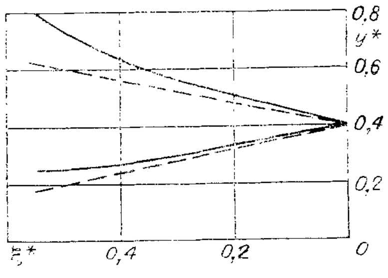

Fig. 1

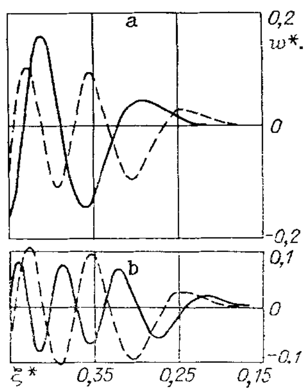

Fig. 2

$$\sigma(y) = \left(\pm\frac{3}{2}N^{-2}\gamma^{-1}\alpha_1(y)\right)^{-1/3}. \quad (4.6)$$

Let us write down the expression for the argument $\varphi(\xi, y)$ of the Airey function derivative

$$\varphi(\xi, y) = \left(\frac{\xi}{V} \mp \frac{(\alpha_1(y) - \alpha_2(y))}{\gamma}\right)\left(\pm\frac{3}{2}\frac{\alpha_1(y)}{N^2\gamma}\right)^{-1/3}. \quad (4.7)$$

Using the Liouville theorem [9], we have the geometric divergence $J = \sqrt{V^2 - \gamma^2 y^2}$.

Therefore, all the elements in the solution for w (3.5) are found. We present the final expression

$$w = \frac{Q\sigma^2(y)\,c^{3/2}(y_0)}{2c^{1/2}(y)}\left(\frac{V^2 - c^2(y_0)}{V^2 - c^2(y)}\right)^{1/4} f_z'(z_0, y_0)\, f(z, y)\,\mathrm{Ai}'(\varphi(\xi, y)) \quad (4.8)$$

[the functions $c(y)$, $f(z, y)$, $\sigma(y)$, and $\varphi(\xi, y)$ are determined from (4.1), (4.6), and (4.7)].

Results of numerical computations in the dimensionless variables $\xi^* = \xi\gamma/V$, $y^* = y\gamma/V$, $z^* = z/\beta y_0$, $Q^* = QN^2/V^3$, $w^* = w/V$ are given in Figs. 1 and 2. The left and right fronts computed by means of (4.5) for $y_0^* = 0.4$ are shown in Fig. 1. The solid lines in Fig. 2 are graphs of the vertical velocity $w^*(\xi^*)$ constructed by means of (4.8) for $Q^* = 1$, $z_0^* = 0.2$, $z^* = 0.1$, and $y^* = 0.29$ (a), $y^* = 0.51$ (b); the dashes are the vertical velocity for the constant depth $H^* = 1$. It is seen that the wave amplitude for a variable bottom is less at the left of the motion axis than for a constant bottom and is greater at the right.

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